Probably Recordings from a Lecture about Methods of Astronomical Observations in the year around 1990 in the Astronomical Institute in Vienna.

The professor reported about overlaped photography of the sky in different time, while all objects move and the mathematics which gives out the orbit parameters of all objects (calculated back to the equinox of 1st Jan 1900 or so), while accuracy errors are corrected especially the distortion of the telescope.

$$x = x_0 + \xi$$

$$\varphi(\xi) = Ce^{\left(-\frac{1}{2}\xi^T\sigma^{-1}\xi\right)}$$

$$\sigma(\varrho, P) = \begin{pmatrix} \sigma_{\varrho\varrho} & 0 \\ 0 & \varrho_{PP} \end{pmatrix}$$

$$\sigma(x, y) = \left(\frac{\vartheta(x, y)}{\vartheta(\varrho, P)}\right) \sigma(\varrho, P) \left(\frac{\vartheta(x, y)}{\vartheta(\varrho, P)}\right)^T$$

$$\frac{\vartheta(x, y)}{\vartheta(\varrho, P)} = \begin{pmatrix} \cos P & \sin P \\ -\varrho \sin P & \varrho \cos P \end{pmatrix}$$

$$\sigma(x, y) = \begin{pmatrix} \sigma_{\varrho\varrho} \cos P & \sigma_{PP} \sin P \\ -\varrho\, \sigma_{\varrho\varrho} \sin P & \varrho\, \sigma_{PP} \cos P \end{pmatrix} \begin{pmatrix} \cos P & -\varrho \sin P \\ \sin P & \varrho \cos P \end{pmatrix}$$

ausmultiplizieren

$$\xi^T\sigma^{-1}\xi = Min\,! \quad Multivariate$$

$$F(x_0 + \xi, a) = 0 \qquad a\begin{pmatrix} b + \beta \\ c \end{pmatrix} \qquad \psi(\beta) = Ke^{\left(-\frac{1}{2}\beta^T\sigma^{-1}\beta\right)}$$

$$b = \begin{matrix} b_1 \\ \vdots \\ b_l \end{matrix} \qquad b = \begin{matrix} b_1 + \beta_{11} \\ b_1 + \beta_{12} \\ b_1 + \beta\lambda_1 \\ \hline b_2 + \beta_{21} \\ b_2 + \beta_2\lambda_1 \\ \hline b_l + \beta_{l1} \\ b_l + \beta_l\lambda_l \end{matrix}$$

$$\sigma = I$$

$$\xi^T\xi = \sum_{\mu=1}^{m} \xi^2\mu = Min\,!$$ L2-Form

Form gleichgenauer unkorrelierter Beob.

$F \equiv A\,.\;a - x = 0$ Linearform

m*n n*1 m*1

$$Aa - x_0 = \xi$$

$$(a^T A^T - x_0^T)(Aa - x_0) = \xi^T \xi$$

$$a^T A^T \underbrace{A}_{m,n} \underbrace{a}_{n,1} - \underbrace{x_0^T A\,a - a^T A^T x_0}_{-2a^T A^T x_0} + x_0^T x_0$$

nr of rows and cols of the matrixes must be correct

$$A^T A = Q$$

$$2A^T A\,a = 2A^T x_0 \qquad A^T A\,a = A^T x_0$$

$\hat{a} = (A^T A)^{-1} A^T x_0$ Bedingungsgleichungen

$$\propto y + \beta z - \gamma = 0$$

$$x = \begin{pmatrix} y_1 \\ z_1 \\ y_2 \\ z_2 \\ \vdots \\ y_n \\ z_n \end{pmatrix} \qquad a \begin{pmatrix} \propto \\ \beta \\ \gamma \end{pmatrix}$$

Helmert 1880

Edwards Deming 1943

1955 Duanel Brown

Ballistics Research Lab Rep II 935

Aberdeen Proving Grounds, MD

Astronomical Journal

Monthly Notices

$$s = \xi^T \sigma^{-1} \xi - 2\Lambda^T F \quad \rightarrow \quad \frac{\vartheta s}{\vartheta \xi}$$

$A = \left(\frac{\vartheta F}{\vartheta a}\right)$ $\quad X = \left(\frac{\vartheta F}{\vartheta x}\right)$ Funktionalmatrix

$$\boxed{\xi = \underbrace{\sigma}_{m*m} \underbrace{X^T}_{m*p} \underbrace{\Lambda}_{p*1}}$$

$$\boxed{F(x_0 + \sigma X^T \Lambda, a) = 0 \qquad \underbrace{A^T}_{n*p} \underbrace{\Lambda}_{p*1} = 0}$$

p+n Gleichungen

$$A = \left(\frac{\vartheta F}{\vartheta a}\right)_{\substack{a=a_0 \\ x=x_0}} \qquad \frac{\|\propto\|}{a} \ll 1, \quad \frac{\|\xi\|}{\|x_0\|} \ll 1$$

$$X = \left(\frac{\vartheta F}{\vartheta x}\right)_{\substack{a=a_0 \\ x=x_0}}$$

$$X\xi + A\alpha + F_0 = 0$$

$$F_0 = F(x_0, a_0) \quad \underbrace{\begin{pmatrix} X_\sigma X^T & A \\ A^T & 0 \end{pmatrix}\begin{pmatrix} \Lambda \\ \alpha \end{pmatrix} + \begin{pmatrix} F_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{\substack{X_\sigma X^T \Lambda + A\alpha + F_0 = 0 \\ \Lambda = -\left(X\sigma X^T\right)^{-1}(A\alpha + F_0)}}$$

$$\alpha = -[A^T(X\sigma X^T)^{-1}A]^{-1}A^T(X\sigma X^T)^{-1}F_0$$

$Aa - x_0 \equiv \xi \qquad \sigma = s\,I$	klassisches Ergebnis Spezialfall von oben

$$A = A \qquad X = -I \qquad X\sigma X^T = s\,I$$

$$(X\sigma X^T)^{-1} = \frac{1}{s}\,I \qquad F_0 =$$

$$\alpha = (A^T A)^{-1} A^T x_0$$

Fälle, daß die Matrix $X\sigma^T\Lambda$ nicht singulär ist

$$(F = \alpha y + \beta z - \gamma = 0)$$

$$X = \begin{matrix} \alpha\beta & 00 & \dots & 00 \\ 00 & \alpha\beta & \dots & 00 \\ \vdots & \vdots & \ddots & \vdots \\ 00 & 00 & \dots & \alpha\beta \end{matrix}$$

$$A = \begin{pmatrix} y_1 z_1 & -1 \\ y_2 z_2 & -1 \\ y_m z_m & -1 \end{pmatrix} \qquad X\sigma X^T = \begin{pmatrix} \alpha^2 + \beta^2 & 0 & \dots & 0 \\ 0 & \alpha^2 + \beta^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \alpha^2 + \beta^2 \end{pmatrix}$$

$$(X\sigma X^T)^{-1} = \frac{1}{\alpha^2 + \beta^2} I_m$$

$$A^T(X\sigma X^T)^{-1}\begin{pmatrix} y_1 & y_2 & \cdots & y_m \\ z_1 & z_2 & \cdots & z_m \\ -1 & -1 & \dots & -1 \end{pmatrix}$$

$$\alpha = -[A^T(X\sigma X^T)^{-1}A]^{-1}A^T(X\sigma X^T)F_0$$

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wenn sie nicht singulär ist

$$F = \begin{pmatrix} G(x,a) \\ H(a) \end{pmatrix} \qquad 0 \qquad \alpha + \beta + \gamma = \Pi$$

$$X = \begin{pmatrix} \left(\frac{\vartheta G}{\vartheta x}\right) \\ 0 \end{pmatrix} \qquad \left(\frac{\vartheta G}{\vartheta x}\right) = y$$

$$X\sigma X^T = \begin{pmatrix} Y\sigma Y^T & 0 \\ 0 & 0 \end{pmatrix}$$

Folgendes:

$$A = \begin{pmatrix} B \\ C \end{pmatrix} = \begin{pmatrix} \left(\frac{\vartheta G}{\vartheta a}\right) \\ \left(\frac{\vartheta H}{\vartheta a}\right) \end{pmatrix} \qquad \Lambda = \begin{pmatrix} M \\ N \end{pmatrix}$$

$$\begin{pmatrix} Y\sigma Y^T & 0 & B \\ 0 & 0 & C \\ B^T & C^T & 0 \end{pmatrix}\begin{pmatrix} M \\ N \\ \alpha \end{pmatrix} + \begin{pmatrix} G_0 \\ H_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

/ singulär, invertierbar

$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ A, D quadratisch u. nicht gleichzeitig singulär

$$|A|^2 + |D|^2 \neq 0 \;\rightarrow\; |A| \neq 0$$

zu invertieren

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}\begin{pmatrix} U & V \\ X & Y \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

$$AU + BX = I$$
$$CU + DX = 0$$

$$U = A^{-1}(I - BX)$$
$$U = A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1}$$

$$(-CA^{-1}B + D)X + CA^{-1} = 0$$
$$X = -(D - CA^{-1}B)^{-1}CA^{-1}$$

$$\left.\begin{array}{l} AU + BX = I \\ CU + DX = 0 \end{array}\right| \quad -BD^{-1},\ addieren$$

$$\underbrace{U = A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} = (A - BD^{-1}C)^{-1}}_{Inversions-Lemma}$$