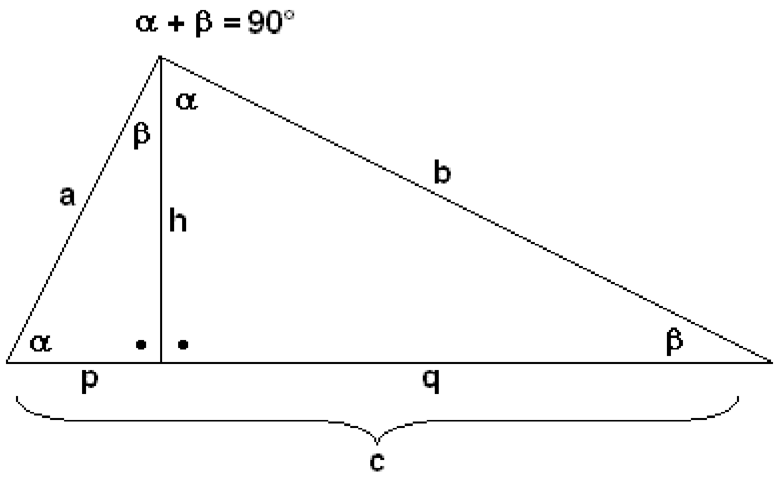
An Evidence of Pythagoras' Theorem

Rectangular Triangle



1. The angle-sum in any triangle is 180°, therefor in a rectangular triangle there is $\beta = 90 - \alpha$
2. By triangles, which differ in size, but have equal angles, the lenght of sides behave proportional to each other. By drawing-in the hight-line one obtains 2 of such triangles (a-h-p und b-q-h).
3. If one draws in any rectangle a diagonal, it is divided by this into 2 equal, rectangular triangles. So every of these triangles has exactly the half area of the rectangle. So one can determine the area of the rectangular triangle, by drawing a rectangle over it.

$$\frac{p}{h} = \frac{h}{q} \quad \Rightarrow \quad h^2 = p \cdot q$$

So one can determine the hight of the triangle, but p and q too, without knowing the angles:

$$\text{A ... Area of the Triangle}$$

$$2 \cdot A = a \cdot b = h \cdot c \qquad \Rightarrow \qquad h = \frac{a \cdot b}{c}$$

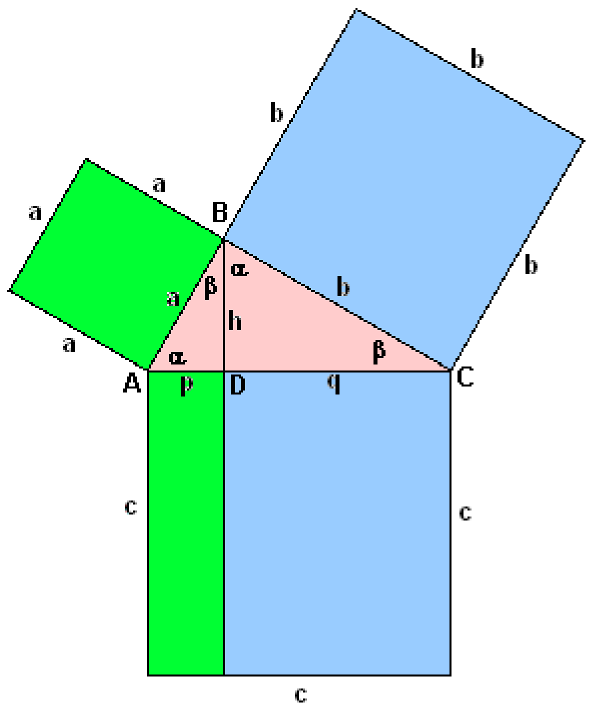
$$\frac{p}{a} = \frac{h}{b} \quad \Rightarrow \quad p = h\frac{a}{b} = \frac{a^2}{c} \qquad\qquad \frac{q}{b} = \frac{h}{a} \quad \Rightarrow \quad q = h\frac{b}{a} = \frac{b^2}{c}$$

$$c = p + q = \frac{a^2}{c} + \frac{b^2}{c} = \frac{a^2 + b^2}{c} \quad \Rightarrow$$

$$\boxed{c^2 = a^2 + b^2}$$

Pythagoras' Theorem

Evidence by comparison of the areas a^2 und $p \cdot c$



In the both triangles of the same angle α between a und p it is:

In the triangle ABD: $p = a \cdot \cos\alpha$

In the triangle ABC: $a = c \cdot \cos\alpha$

$k = \cos\alpha$ one take k instead of the Cosine α as shorteningfactor:

$$p \cdot c = a \cdot k \cdot \frac{a}{k} = a^2$$

after this the green rectangle has the same area as the green square

by the blue it is the same, if one replaces α by β , a by b und p by q. By the so called "projection" of a on p the shortening is by the same factor as by the projection of c on a, because the angle is the same. That means: What has been arisen by the projection of the side of the square by the green rectangle: What the length of the side of p versus a is shortened, the length of the side of c is prolonged by the same factor versus a. Thus is the area of the rectangle equal to the area of the square.

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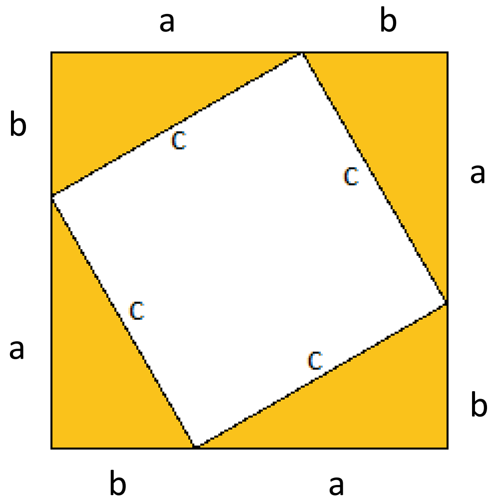
June 2008

The best evidence of Pythagoras' Theorem



A .... Area

$$A_{triangle} = \frac{ab}{2}$$

$$A_{bigsquare} = c^2 + 4.A_{triangle}$$

$$A_{bigsquare} = c^2 + 2.ab = (a+b)^2 = a^2 + 2ab + b^2$$

from it follows : $\mathrm{c}^2 = a^2 + b^2$

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